Power system reliability assessment based on Large Deviation Theory bounds

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***Abstract*— The ability of the power system to meet the expected demand is one of the main issues of electricity system reliability assessment. In the case of the load is greater than the generation capacity the system is at risk, not to be able to serve the demand. The most commonly used reliability measure in electricity power systems is the Loss of Load Probability (LOLP). Our paper introduces a new approach to the assessment of reliability of a power system, which is viewed as its ability to meet the capacity limit. More precisely, we give an upper limit on the probability of overconsumption. Our method needs the bottom-up modelling of the aggregate load of a consumption area and two states Markovian Models are shown to be an adequate tool for this purpose.**

***Index Terms*-- bottom-up modelling, Large Deviation Theory, loss of load probability, power system reliability**

NOMENCLATURE

p	LOLP (Loss of Load Probability).
p_{emp}	Empirical probability of overload.
$\hat{p}$	Upper bound on LOLP.
α	Performance metric (p_{emp} / p).
C	Capacity threshold.
k	Discrete time instant.
$X[k]$	Aggregate consumption at time instant k .
$X_n[k]$	Consumption of the n th appliance at time instant k.
$\overline{X}[k]$	Consumption of the ON/OFF equivalent of the n th appliance at time instant k.
$a_M^{(l)}$	Level of consumption of the M th Markovian state of the l th appliance.
L	Number of appliance classes.
N	Total number of appliances.
M	Number of Markovian states.
p_{ON}	Probability of the ON-state in the case of ON/OFF (two-state) models.
$\mathbf{p}[0]$	Initial state distribution of the Markov chain.
$\tilde{\mathbf{p}}^{(l)}[k]$	State distribution vector of the Markovian model at time instant *k*.
$\tilde{\mathbf{p}}^{(l)}[k]$	State distribution vector of the ON/OFF equivalent model at time instant *k*.
$\mathbf{P}[\mathrm{k}]$	Transition probability matrix.
$P_{ij}[k]$	Element of the transition probability matrix.
$\boldsymbol{\pi}$	Stationary state distribution vector of the Markov chain.
$\hat{U}(X,C)$	Bound on the tail probability of the aggregate consumption.
$\psi_j(s)$	Logarithmic momentum generating function.
s^*	Parameter that satisfies the tightest bound in Chernoff.

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I. INTRODUCTION

In the near future smart metering is forecasted to be ubiquitous [1]. Smart meters are capable to collect statistical information of individual appliances, which gives rise to apply bottom-up aggregate consumption models in practice. As a result, the probability density function (pdf) of the aggregate consumption can be determined from the individual appliance statistics delivered by the smart meters. Additionally, the bottom-up consumption model makes possible to determine Loss of Load Probability (LOLP) as the tail probability of the pdf of the aggregate consumption. Furthermore, Large Deviation Theory (LDT) inequalities can be applied to get an upper bound on the LOLP by a computationally easy way. Hence, sharp LDT bounds on LOLP make us possible to determine the reliability of the network with much less overhead in the case of sizing buses or transformers as before.

There are many reliability indices developed in the power system reliability literature, and these indices can be assessed in different levels of the hierarchy of the power system. For a long time (still the 80's and 90's) mainly deterministic approaches were used and concentrated on the generation side [2]. Recently the research focuses on probability based methods for the generation level, such as methods for computing LOLP using the load-duration curves, daily peak load variation curves and chronological sequence of daily peak loads [3]. Alternatively, these metrics can be calculated from the system margin distribution (convolution of the generation and load distribution functions) [4], [5]. However LOLP is one of the most essential and widely used measure of reliability, there are also more sophisticated measures in the literature, which take into consideration not only the probability but the magnitude, duration and/or frequency of loss as well (e.g. EENS, F&D, etc., see [6] and [7] for more details). Our paper focuses on estimating the LOLP of a single bus or transformer (low-level of the power system hierarchy) based on bottom-up modelling, similarly to [8] and [9], where the transformer sizing problem is addressed based on teletraffic theory and bottom-up modelling of the load, also. The authors of [8] and [9] apply a probabilistic sizing technique originally developed for sizing buffers and links in telecommunications networks to size storage capacity and transformers so that it satisfies LOLP criterion. The main contribution of the present paper is the extension of the Chernoff bound to be able to handle Markovian appliance models.

Mathematically, as mentioned earlier, the LOLP is the upper tail of the probability density function (pdf) of the aggregate consumption of all the appliances connected to a given bus or transformer. The pdf of the aggregate consumption can be estimated from (i) historical aggregate data by different statistical methods, or (ii) can be calculated analytically by the convolution of the individual pdfs of the appliances. In the latter case the stock of appliances and their usage patterns have to be known which is referred to as the 'Capasso model' [10], [11]. The upcoming spread of smart metering motivates us to use this bottom-up modeling approach, since smart meters can measure the consumption of individual appliances and can calculate their individual statistical parameters. (A good reference of different consumption models can be found in [12]). Additionally in this paper we assume that the aggregated consumption time series consists of the sum of individual consumption time series of the appliances.

For large number of appliances, the calculation of the pdf of the aggregate consumption from the pdfs of the individual appliances by convolution may not be computationally feasible as regards storage capacity and time. In this case, (assuming fine-grained appliance models) Large Deviation Theory (LDT) bounds can be used to estimate LOLP as the tail probability of the aggregate pdf. In this paper Bennett's, Hoeffding's and Chernoff's upper-bounds will be applied and compared to the analytical solution. The application of strict upper bounds on the LOLP yields worst-case estimations, i.e. type II error can be excluded.

This paper concentrates on household appliances, however the proposed method is not limited to the residential sector. The statistical parameters of the appliance models are coming from fitting to real measurement data (from freely available data sets). The appliance consumption time series are modelled by Bernoulli iid random variables for mathematical convenience, and by Markovian models for the sake of a more realistic scenario. The appliances are modelled by two consumption levels (ON/OFF) as it is shown to be satisfactory in [8].

The rest of the paper is organized as follows. In Section II we introduce the underlying appliance level consumption models. In Section III we show how LDT inequalities can be applied in a bottom-up model to upper-bound the LOLP. In Section IV the method is extended for Markov models. Section V introduces numerical results and discussion. In Section VI conclusions are drawn.

II. Bottom-up Consumption Time Series Modelling

In traditional electricity networks aggregate consumption time series are modelled at the service providers. The aim is to forecast the load curve of the following day. Smart metering allows us to measure and model consumption in micro level, even on the level of individual appliances. Assuming ubiquitous smart metering technology in the near future a new kind of modeling comes to the fore which is called bottom-up modeling. In the case of bottom-up modelling the aggregate consumption is built-up from the individual consumption models of the appliances. In this paper we use two different appliance-level models: (i) two-state Bernoulli independent identically distributed (iid) random variable sequence – for the sake of mathematical convenience; (ii) discrete time First Order Markov chain (FOM). The basic benefit of the Bernoulli iid model is its simplicity; however, it cannot represent the severe autocorrelation of real consumption time series. Nevertheless, FOM is a more complex model, which is capable to model the autocorrelation as well, and it is shown to be an adequate tool for consumption time series modeling [8], [13], [14]. In the case of Bernoulli iid only two parameters are needed: the probability of the ON-state p_{ON} and the level of the ON-state. (The probability of the OFF-state is $p_{OFF} = 1 - p_{ON}$ and the level is generally zero). In the case of FOM the system is assumed to have finite number of states: $A = \{a_1, \dots, a_M\}$. At time instant k there is a state transition probability matrix $\mathbf{P}[\mathrm{k}]$, where

$$P_{ij}[k] = \Pr\left(X_n[k+1] = a_j \mid \mathrm{X}_n[k] = a_i\right) \quad (1)$$

and $X_n[k]$ denotes the time series of the n th appliance. In this paper we assume stationary Markov chains for which $\mathbf{P}[k] = \mathbf{P}$ stands for all time instances. The initial state distribution

$$\mathbf{p}[0] = \left[\Pr\left(X_n[0] = a_1\right), \dots, \Pr\left(X_n[0] = a_M\right)\right]^T \quad (2)$$

is assumed to be known. Using (1) the state distribution vector at time instant k can be calculated as

$$\mathbf{p}[k] = \mathbf{P}^k \mathbf{p}[0] \tag{3}$$

We assume aperiodic and irreducible Markov chains for which there exist a unique stationary state distribution vector $\boldsymbol{\pi} = \mathbf{P}\boldsymbol{\pi}$.

In all the investigations of this paper the models were fitted to measured data coming from the REDD [15] and GREEND [16] datasets. The REDD dataset contains appliance level power data for 6 homes for several weeks with sampling time of 3 seconds, while GREEND dataset provides 3-6 month appliance level power measurements of 8 buildings (9 sensors/home) with 1Hz. In the context of bottom-up modeling aggregate consumption time series can be built by summing individual time series.

III. Mathematical tools for calculation of the LOLP in the case of Bernoulli iid model

As mentioned in Section II the Smart Meter can determine the individual pdfs of the installed appliances. As a consequence, we can build bottom-up consumption models in the smart meters, or based on aggregating the data of the smart meters, for much larger consumption districts as well. Based on the bottom-up model Large Deviation Theory (LDT) bounds can be used for the estimation of the LOLP.

We use the following mathematical model. There are N appliances that are connected to a smart meter. The consumption of the n th appliance at time instant k is $X_n[k]$. The random variables X_n are assumed to be independent. (Note, that the statistical descriptors of X_n are mostly time dependent, but the random variables are independent, e.g. the probability of turning on the light in House A is correlated with the date and hour of the day, but independent from the behavior of House B). The aggregate consumption at a given time instant k is

$$X[k] = \sum_{n=1}^{N} X_n[k]. \tag{4}$$

Note, that the time dependence will be omitted, if it is not relevant (e.g. in the case of Bernoulli iid sequences).

The LOLP (*p)* is defined [3] as

$$p = \Pr(X \geq C) \tag{5}$$

where C denotes a predefined capacity limit.

In this section our aim is to introduce the mathematical tools by which the LOLP (assuming a given set of appliances connected to a smart meter) for a given capacity limit C, or vice versa the limit C for a given LOLP can be calculated or estimated. Note that only conservative (worst-case) estimations will be investigated; i.e. only upper bounds on the LOLP will be taken into account by which Type II error can be excluded.

A. Exact calculation of the LOLP

Based on definition (5) the LOLP can be seen as the upper tail of the pdf of the aggregate consumption $f_X(x)$. In the case of buttom-up modelling $f_X(x)$ can be determined by the convolution of the individual pdfs of all appliances:

$$f_X(x) = Pr\left(\sum_{i=1}^{N} X_i = x\right) = Pr\left(\sum_{i=1}^{L}\sum_{j=1}^{n_i} X_{ij} = x\right) =$$
$$= f_{X_{11}}(x) * f_{X_{12}}(x) * \cdots * f_{X_{Ln_i}}(x) \tag{6}$$

where all the N appliances connected to a given smart meter are considered to be grouped into L classes. All the appliances in the same class have the same statistical descriptors. The number of appliances in the *i*th class is denoted by n_i. Hence, $N = \sum_{i=1}^{L} n_i$ is the total number of appliances.

B. Estimation of the LOLP

The convolution operation (6) by which the aggregate pdf can be calculated can be very time consuming in the case of high number of appliances (e.g. in the case of an entire district). In this case LDT upper bounds can be used to estimate the LOLP as the upper tail of the pdf of the aggregate consumption. (Note that the Central Limit Theorem (CLT) may not be used for tail probability estimation, because CLT is not an upper bound but an approximation. Furthermore it is known that the CLT approximation is uncertain far from the expected value [17].) The LDT bounds are based on the Markov and Chebisev inequalities. The sharpest bounds are Bennett's, Hoeffding's and Chernoff's upper bounds [18]. The upper bound on the LOLP will be denoted by $\hat{p}$:

$$p = Pr(X \geq C) \leq \hat{U}(X, C) = \hat{p}, \tag{7}$$

where $\hat{U}(X, C)$ denotes one of the bounding techniques on the tail probability.

Using this notion Markov's inequality for non-negative X random variable can be expressed as

$$p \leq \frac{\mu}{C} = \hat{p}. \tag{8}$$

Chebysev's inequality states that

$$C \leq \frac{\sigma}{\sqrt{p}} + \mu. \tag{9}$$

Much tighter upper bounds are developed by Hoeffding, Bennett and Chernoff. Hoeffding's inequality is an exponentially decreasing upper bound, which results in a tighter estimation even far from the expected value compared to Markov's and Chebysev's inequalities. It is also based on the expectation that the X_{ij} random variables are independent and have upper and lower bounds: $x_{ijmin} \leq X_{ij} \leq x_{ijmax}$. From Hoeffding's inequality [19] C can be expressed as:

$$C \le \sqrt{-\frac{1}{2}\ln(p)\sum_i\sum_j\left(x_{ijmax}-x_{ijmin}\right)^2}+\mu \quad (10)$$

Bennett's inequality gives exponentially decreasing upper bound like Hoeffding's, which assumes bounded input random variables $\left|X_{ij}\right| \le x_{max}$, and it is formulated in the following form [20]:

$$p \le \exp\left(-\frac{\sigma^2}{x_{max}^2}h\left(\frac{(C-\mu)x_{max}}{\sigma^2}\right)\right), \quad (11)$$

where $h(u)=(1+u)\log(1+u)-u$. In the case of Benett's inequality C cannot be expressed in closed form, but it can be calculated numerically.

Chernoff's inequality is also an exponentially decreasing upper bound [21]:

$$p \le \exp\left(\sum_{j=1}^{N}\psi_j\left(s^*\right)-s^*\mathrm{C}\right), \quad (12)$$

where $\psi_j(s)=\log E\left\{e^{sX_j}\right\}$ are the logarithmic momentum generating functions, and s^* is the parameter that satisfies the possibly tightest bound. In the case of Chernoff bound, calculation of C requires to solve the following optimization problem:

$$C^*,s^* : \inf_{C,\ s}\left\{\sum_{l=1}^{N}\psi_l(s)-sC\right\} \quad (13)$$

$$s.t. \sum_{l=1}^{N}\psi_l\left(s^*\right)-s^*C^* \le -\gamma, \quad (14)$$

where $\gamma=-\log p$. We can solve this optimization problem with nonlinear optimization methods or in our case with iterative computation (see second type of numerical experiments in Section V.).

IV. Mathematical models for Calculation of the LOLP for First Order Markovian model

In this section we extend the mathematical tools introduced in Section III for the Bernoulli iid appliance consumption models. In the case of FOM appliance-level models there is no tool for calculating the model parameters of the aggregate consumption. However the pdf of the aggregate consumption can be calculated by convolution (see. (6)) assuming that the individual appliances has been already converged to their stationary distributions. On the other hand Chernoff's bound can be applied also in this case. In the following derivation we assume N appliances. The lth appliance can be modelled by its state-transition matrix $\mathbf{P}^{(l)}$ defined in (1). Let us introduce the two-state (ON/OFF) equivalent of this Markov chain for the lth appliance so that both the maximum value ($a_M^{(l)}$) and the expected value ($\mu^{(l)}[k]$) in the kth time slot of the (12) equivalent is the same as for the multi-state Markov chain. Let us denote the state distribution vector of the ON/OFF equivalent by $\tilde{\mathbf{p}}^{(l)}[k]=\left[\tilde{p}_0^{(l)}[k],\tilde{p}_1^{(l)}[k]\right]^T$, where

$$\tilde{p}_1^{(l)}[k]=P\left(X_l^{On-Off}[k]=a_M^{(l)}\right)=\frac{\mu^{(l)}[k]}{a_M^{(l)}}= =\frac{\sum_{i=1}^{M}ip_i^{(l)}[k]}{a_M^{(l)}}=\frac{\sum_{i=1}^{M}iP_{ij}^{(l)}p_j^{(l)}[k-1]}{a_M^{(l)}}, \quad (15)$$

$$\tilde{p}_0^{(l)}[k]=P\left(X_l^{On-Off}[k]=0\right)=1-\frac{\mu^{(l)}[k]}{a_M^{(l)}}= =1-\frac{\sum_{i=1}^{M}ip_i^{(l)}[k]}{a_M^{(l)}}=1-\frac{\sum_{i=1}^{M}iP_{ij}^{(l)}p_j^{(l)}[k-1]}{a_M^{(l)}}, \quad (16)$$

As a result, in every time slot (assuming that the initial distribution of the appliance is known) the distribution vector of the lth appliance can be used for evaluating Chernoff's bound as

$$\Pr\left(\bar{X}[k]>C\right) \le \le \exp\left(\sum_{l=1}^{N}\psi_l\left(s^*[k],k\right)-s^*[k]C^*[k]\right) \quad (17)$$

where $\bar{X}[k]=\sum_{l=1}^{N}X_l^{On-Off}[k]$ and

$$\psi_l(s)=\log\left(1-\frac{\mu^{(l)}[k]}{a_M^{(l)}}+\frac{\mu^{(l)}[k]}{a_M^{(l)}}\exp\left(sa_M^{(l)}\right)\right) \quad (18)$$

is the moment generation function of the lth appliance. This calculation is very time-consuming, however in the case of homogenous and irreducible aperiodic discrete-time Markov Chains the lth appliance model has the a unique $\boldsymbol{\pi}^{(l)}$ stationary distribution, for which $\mathbf{P}^{(l)}\boldsymbol{\pi}^{(l)}=\boldsymbol{\pi}^{(l)}$. In the stationary state the stationary distribution vector of the ON/OFF equivalent can be calculated by the same manner as in (15) and (16). Using the set of the stationary distributions of all the N appliances, Chernoff's bound can be evaluated as follows

$$\Pr\left(\bar{X}_\infty>C\right) \le \exp\left(\sum_{l=1}^{N}\psi_l\left(s_\infty^*\right)-s_\infty^*C\right) \quad (19)$$

where

$$\psi_l(s)=\log\left(1-\frac{\mu_\infty^{(l)}}{a_M^{(l)}}+\frac{\mu_\infty^{(l)}}{a_M^{(l)}}\exp\left(sa_M^{(l)}\right)\right) \quad (20)$$

is the moment generation function of the lth appliance and ∞ indicates the property of stationarity. The extended form of the

Chernoff's bound to FOM aggregates as it is given in (19) can be used for the task to calculate the LOLP given the capacity limit C. Practically, the sizing task is more important: there is a given set of appliances connected to a smart meter (or bus), there is a LOLP to be satisfied ($2.74\cdot10^{-4}$, i.e. one day per ten years) and the capacity limit C has to be found in order to size the system. Unfortunately (19) cannot be rewritten to this form explicitly, but the same optimization problem can be solved numerically as given in (13) and (14).

V. Numerical Results

A. Bernoulli iid model

Our first investigation covered the performance evaluation of the different LDT bounding methods related to the analytic LOLP calculation in the case of Bernoulli iid appliance model. The following performance metric will be used:

$$\alpha = \frac{p_{emp}}{p} \quad (21)$$

where p_{emp} is the empirical probability of overload. In our first type of experiments we determine the allowable number of appliances so that the predefined LOLP (p) parameter can be satisfied. In Figure 1 and 2 the empirical probability of overconsumption is depicted as the function of the LOLP. The probability of ON-states (p_{ON}) are derived from real measurements (REDD and GREEND datasets). The experiments cover a broad range of p_{ON} values, ($p_{ON} = 0.012702 \dots 0.33994$), coming from washer-dryer and lighting measurements, respectively. The results are depicted in Figure 1 and 2.

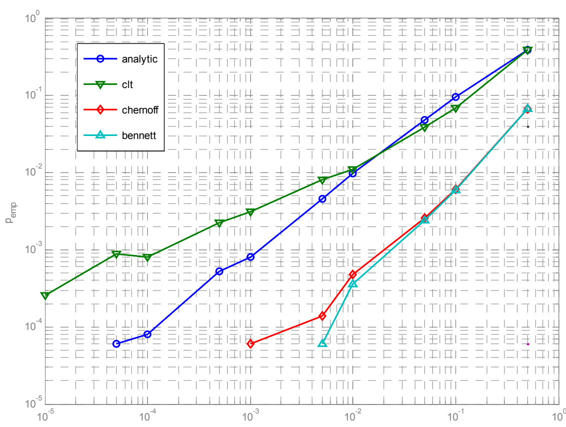

Figure 1. QoS vs empirical probability of overconsumption in the case of Bernoulli iid model with $p_{ON} = 0.012702$

The empirical probability can almost meet LOLP ($\alpha = 1$) in the reference case of analytical calculation of LOLP using the aggregate pdf as it is expressed in (6). There is a small deviation ($\alpha = 0,4 \dots 0,6$) in the range of small probabilities ($10^{-5} \dots 10^{-4}$) due to the difficulty of measuring rare events in the case of Monte Carlo simulations. Using Chernoff's and Bennett's inequalities the ratio α is set to one order of magnitude lower regardless of p_{ON}. Applying Hoeffding's inequality leads to results which highly depend on the p_{ON} value. (Applying Chebisev's and Markov's inequalities lead to poor results regardless of the p_{ON} values.) The performance of CLT is close to the analytic pdf calculation ($\alpha = 1 \dots 3$), but should note that CLT is not an upper bound on the tail probability. As a consequence $\alpha > 1$ values can occur that can cause breach of contract or Type II error.

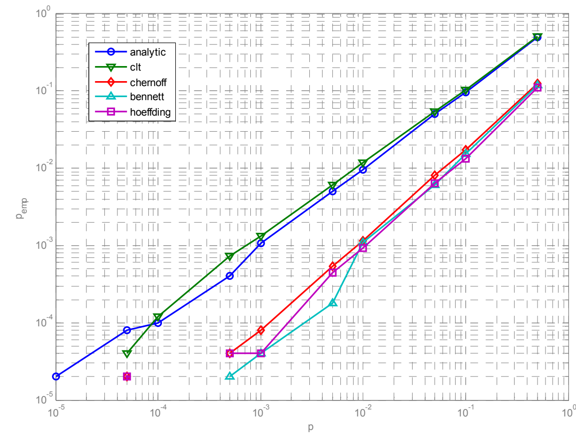

Figure 2. QoS vs empirical probability of overconsumption in the case of Bernoulli iid model with p_{ON} = 0.33994

In our second type of numerical experiments we calculated the capacity limit C in the case of fixed LOLP ($2.74\cdot10^{-4}$). Table I contains the results in the case of Bernoulli iid model, using 5 different types of appliances, ON-state values are set to 1 for all the appliances allowing us to compare the results. The numbers of appliances were set to have the same expected values for each classes. The reference is the result given by the analytical LOLP calculation. The top part of both tables contains the number of appliances, On-state probabilities and exact C values (for each type of calculation methods). For instance, in the case of the washer-dryer, 431 appliances were modelled and summed up.

TABLE I. Bernoulli IID Model

	Appliance Type				
	washer dryer	**microwave oven**	**dishwasher**	**refrigerator**	**lighting**
# of Apps	431	313	113	19	14
ON-state probability	0.012	0.016	0.044	0.254	0.335
analytic	15.0	15.0	15.0	12.8	11.7
Chernoff	17.0	17.0	16.0	13.8	12.7
Bennett	17.0	17.0	17.0	15.8	14.7
CLT	13.0	13.0	13.0	11.8	11.7
Hoeffding	47.0	40.8	26.5	13.6	12.3

Using the analytical aggregate pdf's tail probability, capacity limit $C = 15$ can satisfy the $LOLP = 2.74 \cdot 10^{-4}$. Chernoff's and Bennett's bounds result in $C = 17$, which is only a slight overestimation of the analytic value. This stands for all the appliance types. Hoeffding's bound estimation is highly dependent on p_{ON} (the higher the p_{ON} value the better the bound is). Note that CLT is proven to underestimate the analytical calculation; hence, it cannot guarantee the LOLP (see Section III. B.).

B. First Order Markovian model

We repeated our experiments regarding the calculation of the C with fixed p in the case of FOM models. The state transition matrices were fitted to the time series from the REDD and GREEND data sets. Comparing the results of Table II with the iid version in Table I, we see that actual C values of iid results are higher than the results of FOM. It means that iid modelling is more conservative than FOM, hence, it does not cause Tpye II error. On the other hand FOM gives more realistic time series models, because it introduces time dependence. This feature is important in practical problems like demand side management.

TABLE II. FOM MODEL

	Appliance Type				
	washer dryer	**microwave oven**	**dishwasher**	**refrigerator**	**lighting**
# of Apps	431	313	113	19	14
ON-state stationary probability	0.012	0.014	0.040	0.266	0.313
analytic	14.0	13.5	13.5	12.1	11.4
Chernoff	17.0	15.5	15.5	14.1	12.4
Bennett	17.0	15.5	15.5	16.1	14.4
CLT	13.0	12.5	11.5	12.1	10.4
Hoeffding	41.2	40.4	23.0	13.9	12.0

VI. CONCLUSION

In this paper, we proposed an analytical method to estimate LOLP and to calculate the capacity limit (satisfying LOLP) based on Large Deviation Theory and bottom-up modelling of appliances. We formulated the calculation of capacity limit for the LDT bounds and defined the optimization problem in the case of Chernoff's inequality for both the Bernoulli iid and FOM models. Simulations showed that Chernoff's inequality gives the sharpest upper bound on LOLP, hence it can be used in proper sizing of transformers and buses in the power system. Bernoulli iid and FOM appliance models were tested and our numerical simulations prove that models of the individual appliances have influence on the calculation of LOLP, so our plans are to extend our load models to more realistic ones, which represent time dependence (Higher Order Markov and semi-Markov) as well.

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